Z-boson $p_\perp$ in a virtuality ordered shower

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work with Zoltan Nagy, DESY

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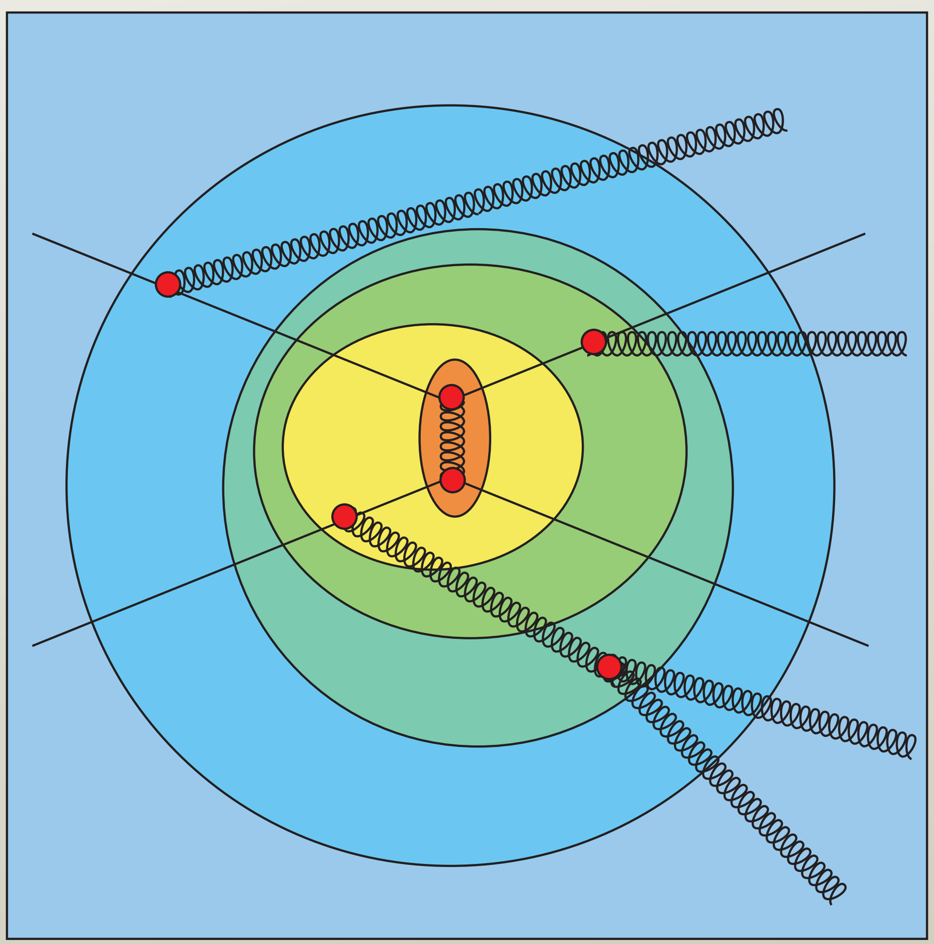
Parton shower evolution

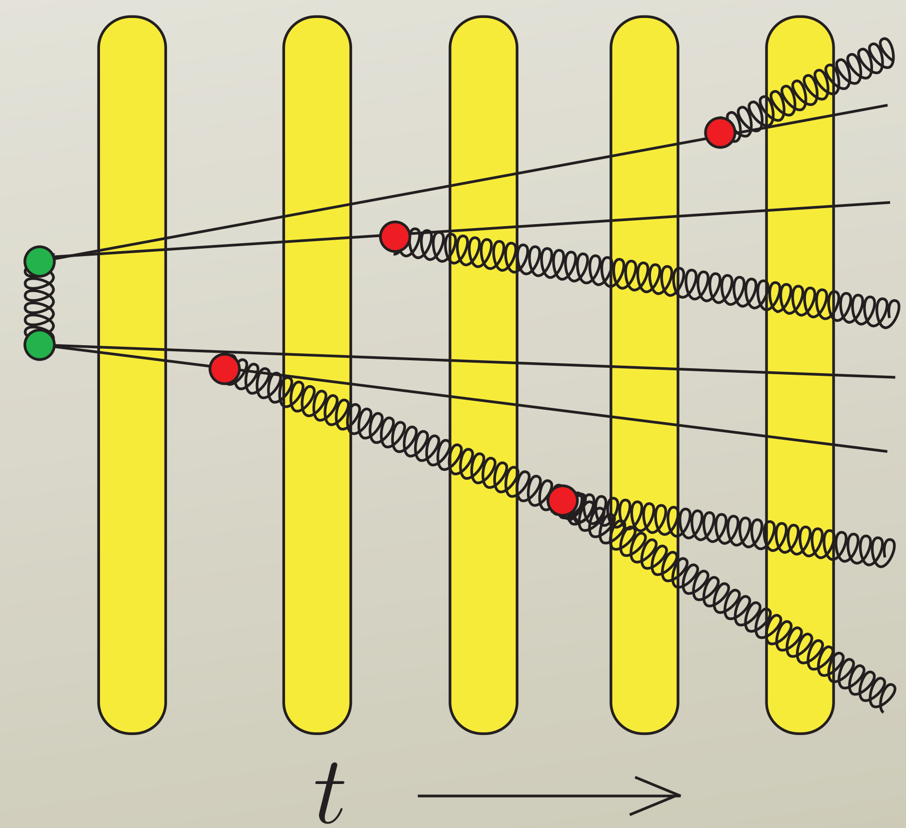
- For this talk, we need to understand what a parton shower does.
- We need not the computer code, but an evolution equation that is implemented by the computer code.
- There are many choices, not all of which can be described by a precise evolution equation.
- I describe a virtuality ordered shower of the type that Zoltan Nagy (DESY) and I are working on.
- I take the spin averaged, leading color version.

The evolution time

- Showers develop in “shower time.”
- Hardest interactions first.
- $t = \log(Q_0^2/Q^2)$, where Q^2 is virtuality of splitting.

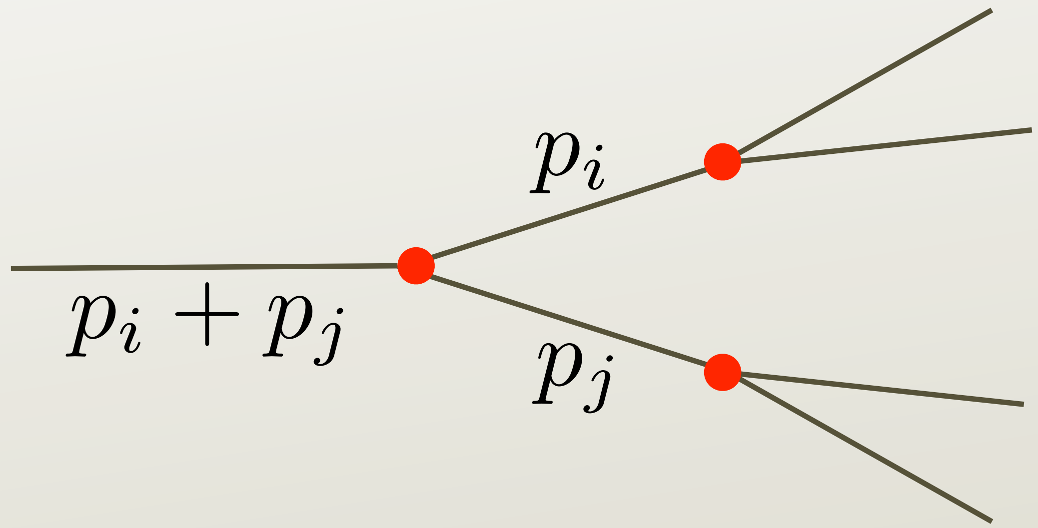


Real time picture



Shower time picture

Why virtuality?



$$\frac{1}{(p_i + p_j)^2} = \frac{1}{2p_i \cdot p_j + p_i^2 + p_j^2} \approx \frac{1}{2p_i \cdot p_j}$$

if $p_i^2 \ll 2p_i \cdot p_j$ and $p_j^2 \ll 2p_i \cdot p_j$.

Statistical states

- Let $\rho(\{p,f,c\}_m)$ be the probability to have m final state partons (plus two initial state partons) with designated momenta, flavors, and colors.
- The state $\big|\rho\big)$ corresponds to the function ρ.
- The state evolves: $\big|\rho(t)\big)$.
- Use basis vectors $\big(\{p,f,c\}_m\big|$.
- $\rho(\{p,f,c\}_m) = \big(\{p,f,c\}_m\big|\rho\big)$.

Measurement functions

- Define a measurement function F as a bra vector.
- Result of measurement for partonic state $\big|\{p,f,c\}_m\big)$ is

$$\big(F\big|\{p,f,c\}_m\big)$$

- Vector corresponding to completely inclusive measurement is $\big(1\big|$:

$$\big(1\big|\{p,f,c\}_m\big) = 1$$

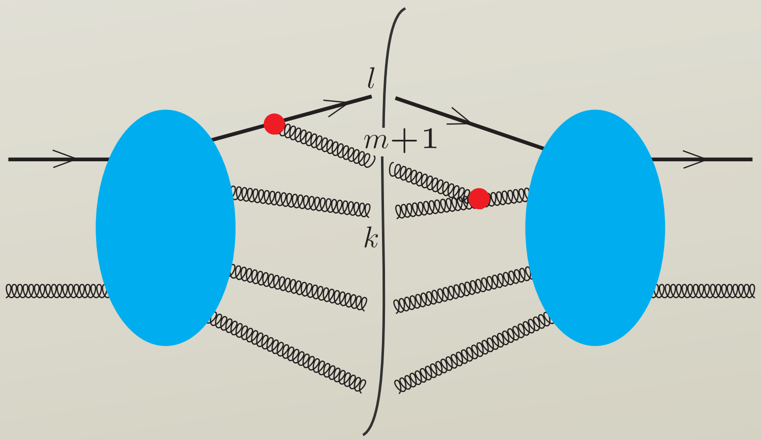
Evolution equation

The shower state evolves in shower time.

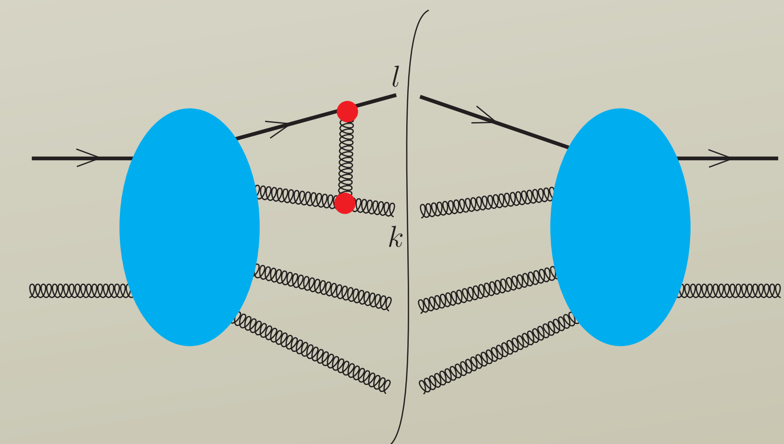
$$\big|\rho(t)\big) = \mathcal{U}(t,t')\big|\rho(t')\big)$$

$$\frac{d}{dt}\,\mathcal{U}(t,t') = [\mathcal{H}_{\mathrm{I}}(t) - \mathcal{V}(t)]\,\mathcal{U}(t,t')$$

$\mathcal{H}_{\mathrm{I}}(t) = \text{real splitting operator}$



$\mathcal{V}(t) = \text{virtual splitting operator}$



Probability conservation

- Evolution does not change the cross section:

$$(1|\mathcal{U}(t,t') = (1|$$

- Since

$$\frac{d}{dt}\mathcal{U}(t,t') = [\mathcal{H}_{\rm I}(t) - \mathcal{V}(t)]\,\mathcal{U}(t,t')$$

- this implies

$$\big(1\big|[\mathcal{H}_{\rm I}(t) - \mathcal{V}(t)] = 0$$

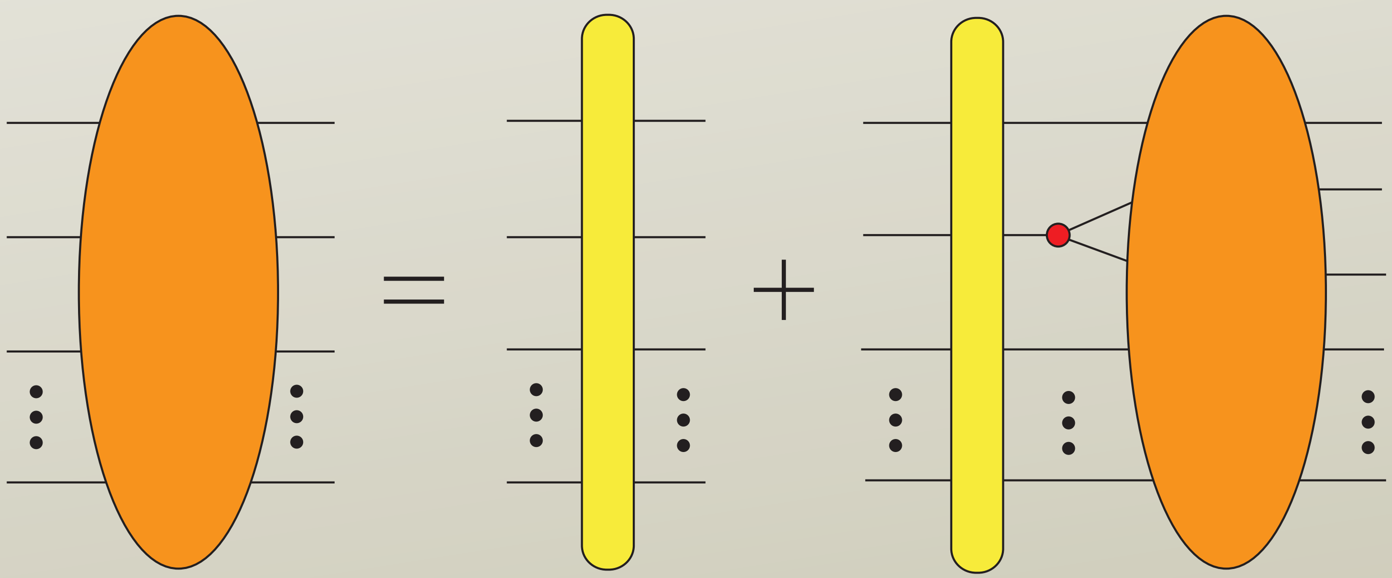
- This suffices to determine $\mathcal{V}$ from $\mathcal{H}_{\rm I}$.

Shower form of evolution

$$\mathcal{U}(t_3,t_1) = \mathcal{N}(t_3,t_1) + \int_{t_1}^{t_3} dt_2\, \mathcal{U}(t_3,t_2)\,\mathcal{H}_{\rm I}(t_2)\,\mathcal{N}(t_2,t_1)$$

Here $\mathcal{N}$ is the Sudakov exponential,

$$\mathcal{N}(t,t') = \mathbb{T}\exp\left\{-\int_{t'}^{t} d\tau\, \mathcal{V}(\tau)\right\}$$



The Sudakov factor represents the probability not to split.

An obvious question

- Is this going to sum large logarithms?

Yes

- The splitting probabilities have the right soft and collinear singularities.
- Parton splitting is iterated.
- So how could it fail?

No

- It has been known since the 1980s that exponentiation of double logs comes from emissions ordered in angles.
- The angle ordering comes from quantum coherence.
- So you need a shower ordered in angles, not virtuality.
- The virtuality ordered shower is doomed.

Logarithms of $p_\perp$

- Consider $A + B \to Z + X$
- Measure the $p_\perp$ of the Z-boson for $p_\perp^2 \ll M_Z^2$,

$$\frac{d\sigma}{dp_\perp dY}$$

- There are large logarithms $\log(M_Z^2/p_\perp^2)$.
- We know how to sum these in QCD.

The QCD answer,

$$
\begin{aligned}
\frac{d\sigma}{d\boldsymbol{p}_\perp dY} \approx \int \frac{d^2\boldsymbol{b}}{(2\pi)^2}\, e^{\mathrm{i}\boldsymbol{b}\cdot\boldsymbol{p}_\perp}
&\times \sum_{a,b} \int_{x_\mathrm{a}}^1 \frac{d\eta_\mathrm{a}}{\eta_\mathrm{a}} \int_{x_\mathrm{b}}^1 \frac{d\eta_\mathrm{b}}{\eta_\mathrm{b}}\; f_{a/A}(\eta_\mathrm{a}, C^2/\boldsymbol{b}^2)\; f_{b/B}(\eta_\mathrm{b}, C^2/\boldsymbol{b}^2) \\
&\times \exp\left(- \int_{C^2/\boldsymbol{b}^2}^{M^2} \frac{d\boldsymbol{k}_\perp^2}{\boldsymbol{k}_\perp^2} \left[A(\alpha_\mathrm{s}(\boldsymbol{k}_\perp^2)) \log\left(\frac{M^2}{\boldsymbol{k}_\perp^2}\right) + B(\alpha_\mathrm{s}(\boldsymbol{k}_\perp^2)) \right] \right) \\
&\times \sum_{a',b'} H^{(0)}_{a'b'}\; C_{a'a}\left(\frac{x_\mathrm{a}}{\eta_\mathrm{a}}, \alpha_\mathrm{s}\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) C_{b'b}\left(\frac{x_\mathrm{b}}{\eta_\mathrm{b}}, \alpha_\mathrm{s}\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) \;.
\end{aligned}
$$

$$
A(\alpha_\mathrm{s}) = 2\, C_\mathrm{F}\, \frac{\alpha_\mathrm{s}}{2\pi} + 2\, C_\mathrm{F}\, \left\{ C_\mathrm{A} \left[\frac{67}{18} - \frac{\pi^2}{6} \right] - \frac{5\, n_\mathrm{f}}{9} \right\} \left(\frac{\alpha_\mathrm{s}}{2\pi}\right)^2 + \cdots \quad ,
$$

$$
B(\alpha_\mathrm{s}) = -4\, \frac{\alpha_\mathrm{s}}{2\pi} + \left[-\frac{197}{3} + \frac{34 n_\mathrm{f}}{9} + \frac{20\pi^2}{3} - \frac{8 n_\mathrm{f} \pi^2}{27} + \frac{8\zeta(3)}{3} \right] \left(\frac{\alpha_\mathrm{s}}{2\pi}\right)^2 + \cdots \quad ,
$$

$$
C_{a'a}(z, \alpha_\mathrm{s}) = \delta_{a'a}\delta(1-z) + \frac{\alpha_\mathrm{s}}{2\pi} \left[\delta_{a'a} \left\{ \frac{4}{3}\,(1-z) + \frac{2}{3}\,\delta(1-z)\,(\pi^2 - 8) \right\} + \delta_{ag}\, z(1-z) \right]
$$

$$
x_\mathrm{A} = \sqrt{\frac{M^2}{s}} e^{Y} \qquad x_\mathrm{B} = \sqrt{\frac{M^2}{s}} e^{-Y} \qquad C = 2 e^{-\gamma_E}
$$

- The most important part is the exponentiation in b-space.
- In exponent,

$$\text{not}\;\; \alpha_{\rm s}(M^2)^n \log(\boldsymbol{b}^2 M^2)^{2n}$$

$$\text{but}\;\; \alpha_{\rm s}(M^2)^n \log(\boldsymbol{b}^2 M^2)^{n+1}$$

$$\begin{aligned}
\frac{d\sigma}{d\boldsymbol{p}_\perp dY} \approx & \int \frac{d^2\boldsymbol{b}}{(2\pi)^2}\, e^{i\boldsymbol{b}\cdot\boldsymbol{p}_\perp} \\
& \times \sum_{a,b} \int_{x_{\rm a}}^{1} \frac{d\eta_{\rm a}}{\eta_{\rm a}} \int_{x_{\rm b}}^{1} \frac{d\eta_{\rm b}}{\eta_{\rm b}}\, f_{a/A}\big(\eta_{\rm a}, C^2/\boldsymbol{b}^2\big)\, f_{b/B}\big(\eta_{\rm b}, C^2/\boldsymbol{b}^2\big) \\
& \times \exp\left(-\int_{C^2/\boldsymbol{b}^2}^{M^2} \frac{d\boldsymbol{k}_\perp^2}{\boldsymbol{k}_\perp^2}\left[A(\alpha_{\rm s}(\boldsymbol{k}_\perp^2)) \log\left(\frac{M^2}{\boldsymbol{k}_\perp^2}\right) + B(\alpha_{\rm s}(\boldsymbol{k}_\perp^2))\right]\right) \\
& \times \sum_{a',b'} H^{(0)}_{a'b'}\, C_{a'a}\left(\frac{x_{\rm a}}{\eta_{\rm a}}, \alpha_{\rm s}\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) C_{b'b}\left(\frac{x_{\rm b}}{\eta_{\rm b}}, \alpha_{\rm s}\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) \quad .
\end{aligned}$$

What we might hope for,

$$
\frac{d\sigma}{d\boldsymbol{p}_\perp dY} \approx \int \frac{d^2\boldsymbol{b}}{(2\pi)^2}\, e^{i\boldsymbol{b}\cdot\boldsymbol{p}_\perp}
$$

$$
\times \sum_{a,b} \int_{x_\mathrm{a}}^1 \frac{d\eta_\mathrm{a}}{\eta_\mathrm{a}} \int_{x_\mathrm{b}}^1 \frac{d\eta_\mathrm{b}}{\eta_\mathrm{b}}\; f_{a/A}(\eta_\mathrm{a}, C^2/\boldsymbol{b}^2)\; f_{b/B}(\eta_\mathrm{b}, C^2/\boldsymbol{b}^2)
$$

$$
\times \exp\left(- \int_{C^2/\boldsymbol{b}^2}^{M^2} \frac{d\boldsymbol{k}_\perp^2}{\boldsymbol{k}_\perp^2} \left[A(\alpha_\mathrm{s}(\boldsymbol{k}_\perp^2)) \log\left(\frac{M^2}{\boldsymbol{k}_\perp^2}\right) + B(\alpha_\mathrm{s}(\boldsymbol{k}_\perp^2)) \right] \right)
$$

$$
\times \sum_{a',b'} H^{(0)}_{a'b'}\; C_{a'a}\left(\frac{x_\mathrm{a}}{\eta_\mathrm{a}}, \alpha_\mathrm{s}\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) C_{b'b}\left(\frac{x_\mathrm{b}}{\eta_\mathrm{b}}, \alpha_\mathrm{s}\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) \;.
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$$

$$
B(\alpha_\mathrm{s}) = -4\,\frac{\alpha_\mathrm{s}}{2\pi} + \cancel{\left[-\frac{197}{3} + \frac{34 n_\mathrm{f}}{9} + \frac{20\pi^2}{3} - \frac{8 n_\mathrm{f}\pi^2}{27} + \frac{8\zeta(3)}{3}\right]\left(\frac{\alpha_\mathrm{s}}{2\pi}\right)^2} + \cdots \;,
$$

$$
C_{a'a}(z,\alpha_\mathrm{s}) = \delta_{a'a}\delta(1-z) + \cancel{\frac{\alpha_\mathrm{s}}{2\pi}\left[\delta_{a'a}\left\{\frac{4}{3}(1-z) + \frac{2}{3}\delta(1-z)(\pi^2-8)\right\} + \delta_{ag}\, z(1-z)\right]}
$$

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x_\mathrm{A} = \sqrt{\frac{M^2}{s}}e^{Y} \qquad x_\mathrm{B} = \sqrt{\frac{M^2}{s}}e^{-Y} \qquad C = 2e^{-\gamma_E}
$$

Analytical approach

- Start with the Fourier transform of the cross section.

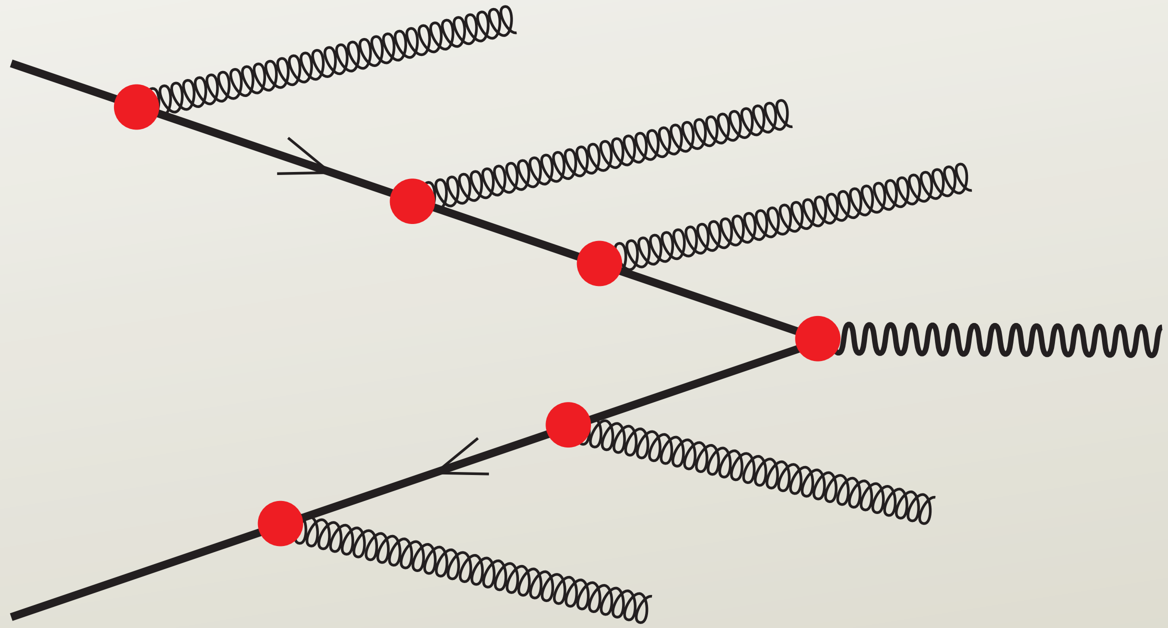
$$\big(\boldsymbol{b}, Y\big|\rho(t)\big) = \int \frac{d\boldsymbol{p}_\perp}{(2\pi)^2}\, e^{i\boldsymbol{p}_\perp\cdot\boldsymbol{b}}\big(\boldsymbol{p}_\perp, Y\big|\rho(t)\big)$$

- Use the shower evolution equation.

$$\frac{d}{dt}\big(\boldsymbol{b}, Y\big|\rho(t)\big) = \big(\boldsymbol{b}, Y\big|\mathcal{H}_{\mathrm{I}}(t) - \mathcal{V}(t)\big|\rho(t)\big)$$

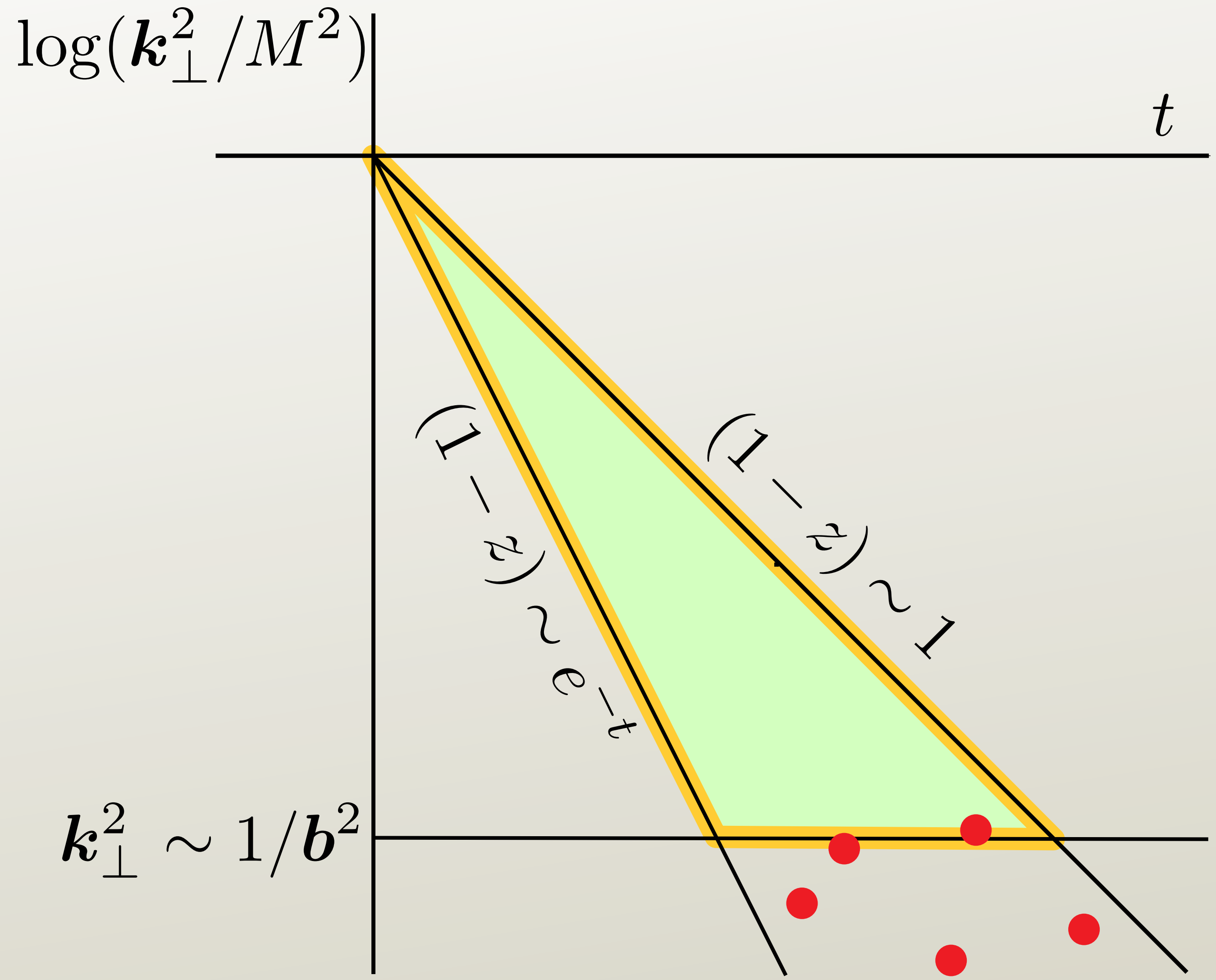
- Use what we know about the operators involved.

The basic physics



- The Z-boson gets transverse momentum because of recoils against initial state radiation. (Parisi & Petronzio)

- There is a certain region of possible emissions.



- Only emissions with $\boldsymbol{k}_\perp^2 < 1/\boldsymbol{b}^2$ allow $\big(\boldsymbol{b}, Y\big|\rho(t)\big)$ to remain non-zero.
- The probability not to emit in the shaded triangle is the Sudakov exponential.

Result

$$\frac{d\sigma}{d\boldsymbol{p}_\perp dY} \approx \int \frac{d^2\boldsymbol{b}}{(2\pi)^2}\, e^{i\boldsymbol{b}\cdot\boldsymbol{p}_\perp}$$

$$\times \sum_{a,b} \int_{x_\mathrm{a}}^{1} \frac{d\eta_\mathrm{a}}{\eta_\mathrm{a}} \int_{x_\mathrm{b}}^{1} \frac{d\eta_\mathrm{b}}{\eta_\mathrm{b}}\; f_{a/A}(\eta_\mathrm{a}, C^2/\boldsymbol{b}^2)\; f_{b/B}(\eta_\mathrm{b}, C^2/\boldsymbol{b}^2)$$

$$\times \exp\left(- \int_{C^2/\boldsymbol{b}^2}^{M^2} \frac{d\boldsymbol{k}_\perp^2}{\boldsymbol{k}_\perp^2} \left[A(\alpha_\mathrm{s}(\boldsymbol{k}_\perp^2)) \log\left(\frac{M^2}{\boldsymbol{k}_\perp^2}\right) + B(\alpha_\mathrm{s}(\boldsymbol{k}_\perp^2)) \right] \right)$$

$$\times \sum_{a',b'} H^{(0)}_{a'b'}\; C_{a'a}\left(\frac{x_\mathrm{a}}{\eta_\mathrm{a}}, \alpha_\mathrm{s}\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) C_{b'b}\left(\frac{x_\mathrm{b}}{\eta_\mathrm{b}}, \alpha_\mathrm{s}\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) \;.$$

✓ Exponentiation

✓ ✓ ✓

$$A(\alpha_\mathrm{s}) = 2\,C_\mathrm{F}\,\frac{\alpha_\mathrm{s}}{2\pi} + 2\,C_\mathrm{F}\left\{C_\mathrm{A}\left[\frac{67}{18} - \frac{\pi^2}{6}\right] - \frac{5\,n_\mathrm{f}}{9}\right\}\left(\frac{\alpha_\mathrm{s}}{2\pi}\right)^2 + \cdots \;,$$

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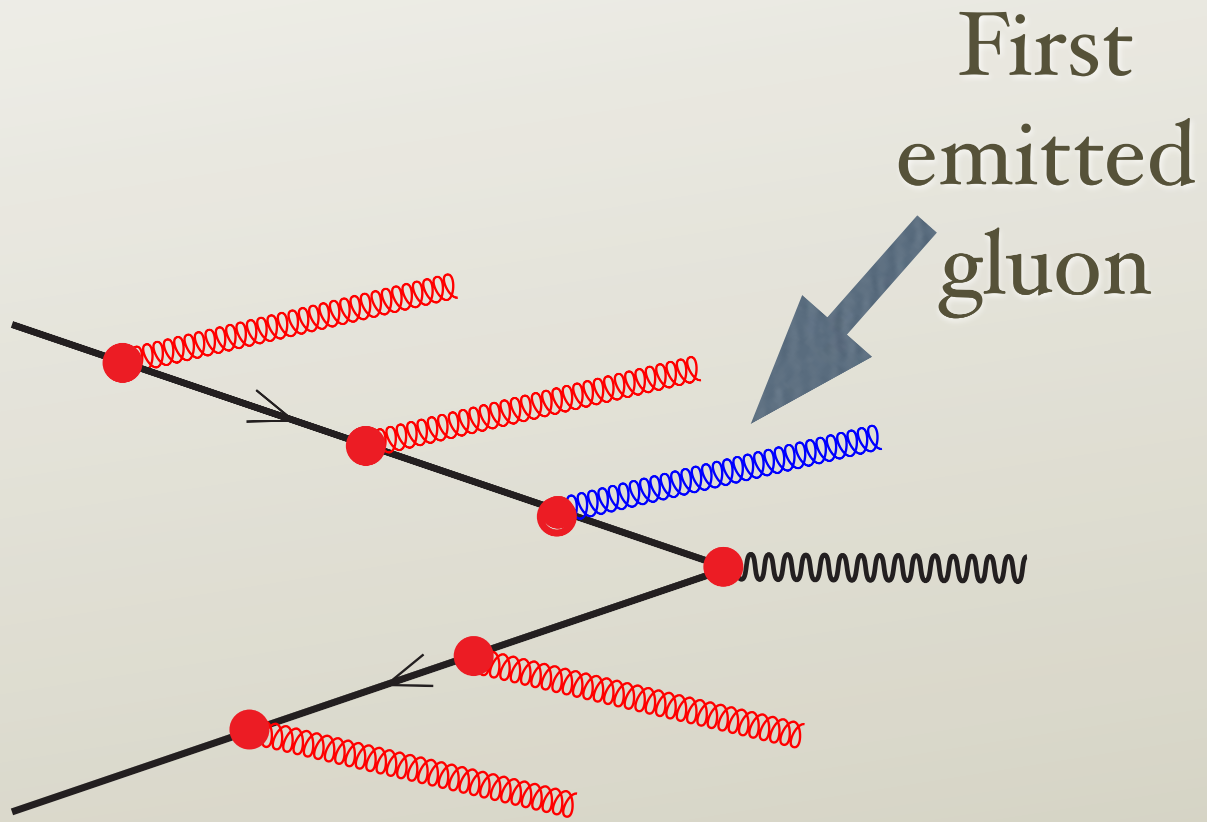
$$C_{a'a}(z, \alpha_\mathrm{s}) = \delta_{a'a}\delta(1-z) + \frac{\alpha_\mathrm{s}}{2\pi}\cancel{\left[\delta_{a'a}\left\{\frac{4}{3}(1-z) + \frac{2}{3}\delta(1-z)(\pi^2-8)\right\} + \delta_{ag}\, z(1-z)\right]}$$

Variations

- How about other types of showers that could be obtained from our virtuality ordered shower by a simple modification?
    - Catani-Seymour dipole shower.
    - Angle ordered shower.
    - k_T ordered shower.
- Note: comments may not apply to any existing shower code.

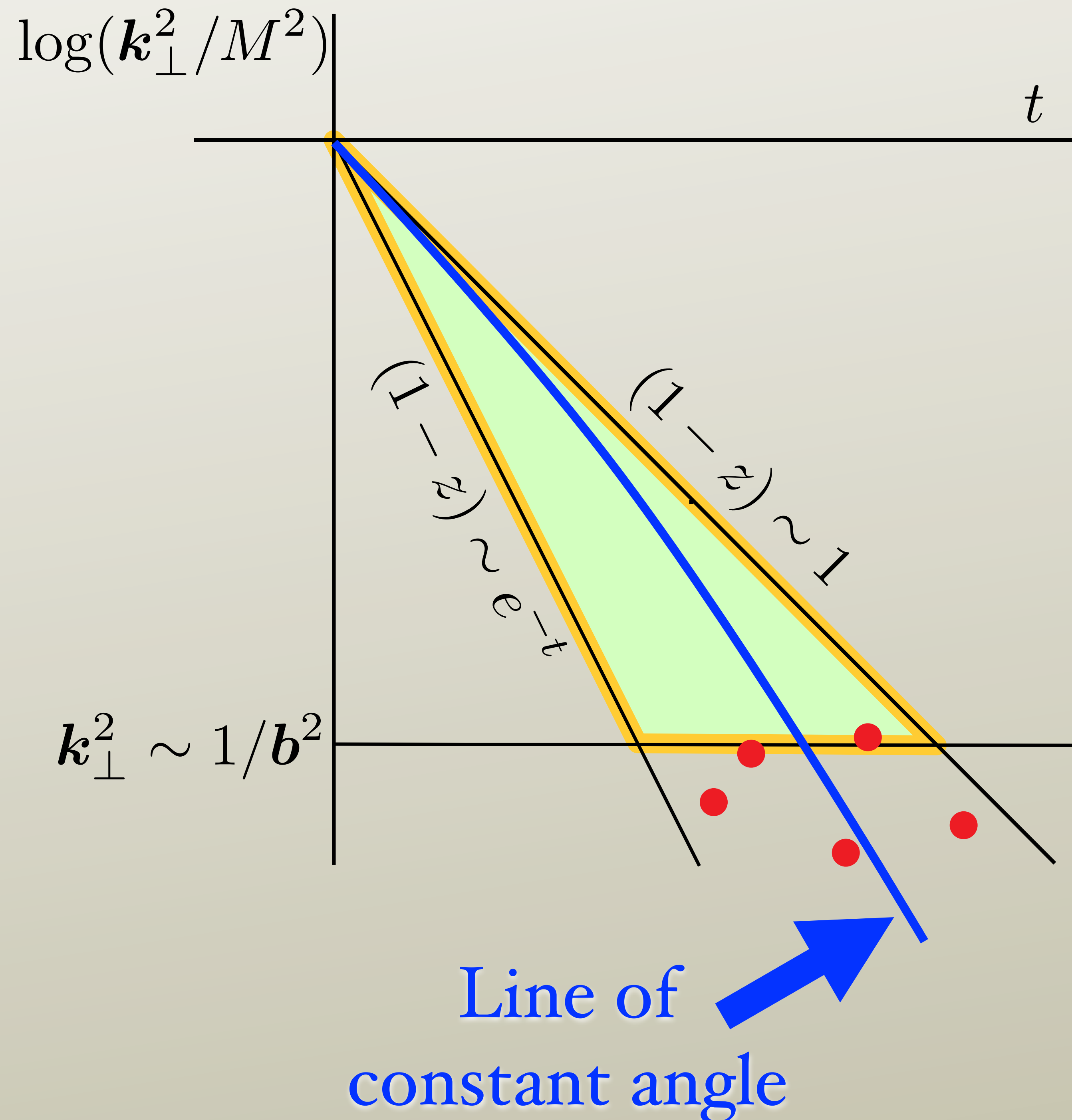
Catani-Seymour dipole shower

- Use momentum mapping of Catani-Seymour dipole scheme.
- Z-boson gets recoil from first emitted gluon.
- Recoil from gluons emitted later is absorbed by gluons already emitted.
- This spoils the summation.



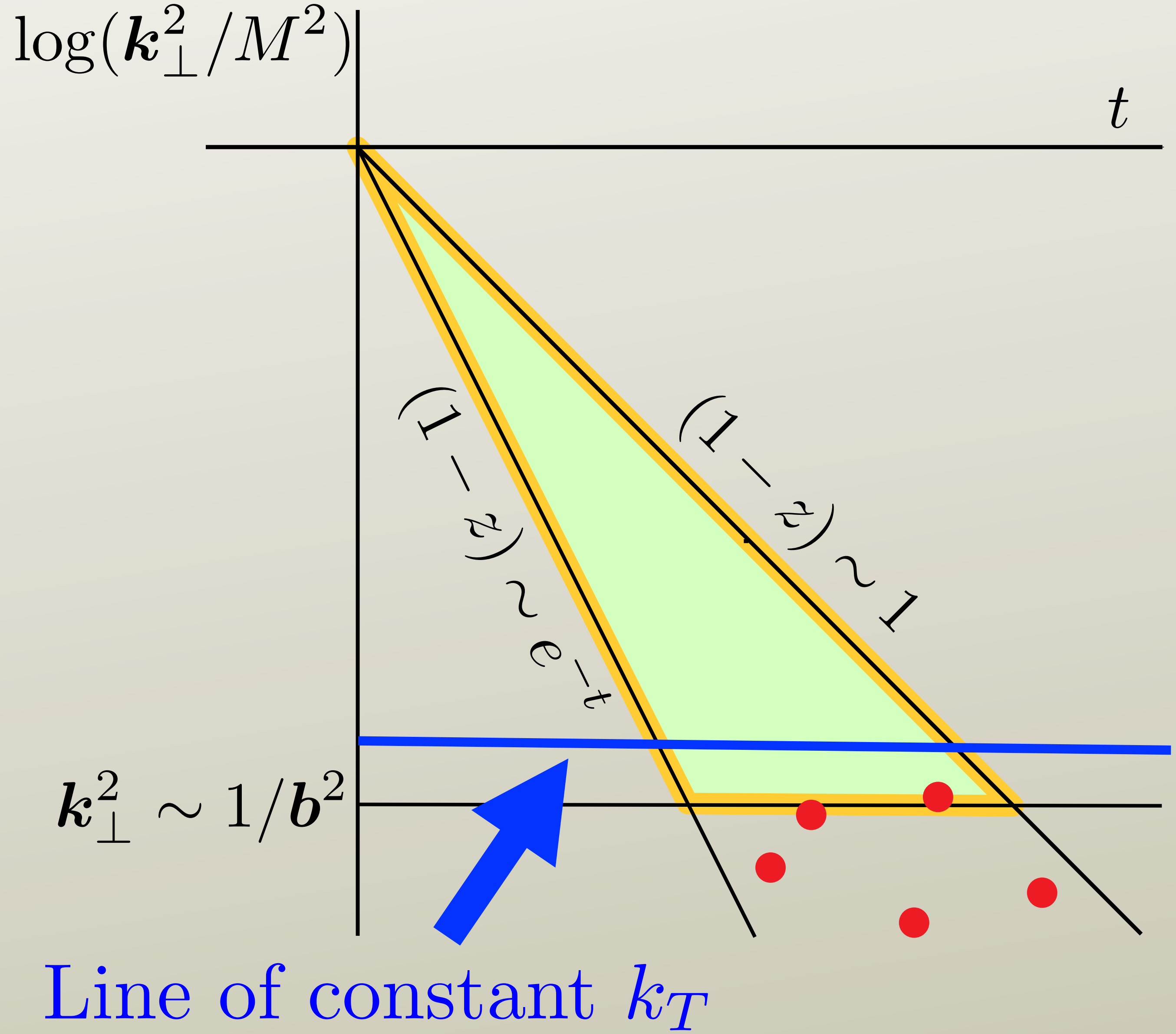
Angle ordered shower

- Use angle instead of virtuality as ordering parameter.
- This works fine.
- In our derivation, we used the fact that smaller t and smaller $k_\perp$ implies larger angle.



k_T ordered shower

- We don't know what happens when k_T^2 has decreased to $k_T^2 \sim 1/b^2$.



Conclusions

- Summation of large logarithms for a given process with a given parton shower algorithm is not obvious.
- Things can go wrong.
- It should be proved analytically.
- I might guess that no parton shower algorithm gets everything right.
- The virtuality ordered shower used here gets one thing right.